

PHYSICS

Paper - I

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are EIGHT questions in all, out of which **FIVE** are to be attempted.

Questions no. 1 and 5 are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections **A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Answers must be written in **ENGLISH** only.

Useful Constants :

Electron charge (e)	$= 1.602 \times 10^{-19}$ C
Electron rest mass (m_e)	$= 9.109 \times 10^{-31}$ kg
Proton mass (m_p)	$= 1.672 \times 10^{-27}$ kg
Vacuum permittivity (ϵ_0)	$= 8.854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 4\pi \times 10^{-7}$ T m/A
Velocity of light in free space (c)	$= 3 \times 10^8$ m/s
Boltzmann constant (k)	$= 1.380 \times 10^{-23}$ J/K
Electronvolt (eV)	$= 1.602 \times 10^{-19}$ J
Planck constant (h)	$= 6.626 \times 10^{-34}$ J s
Stefan constant (σ)	$= 5.67 \times 10^{-8}$ W m $^{-2}$ K $^{-4}$
Avogadro number (N)	$= 6.022 \times 10^{26}$ kmol $^{-1}$
Gas constant (R)	$= 8.31 \times 10^3$ J kmol $^{-1}$ K $^{-1}$
exp (1)	$= 2.718$

SECTION A

Q1. (a) Two particles each of rest mass 1 gm collide head-on with the speed $0.8c$ to stick together at rest. Find the mass of the final composite. 8

(b) Calculate the effective gravitational acceleration at the equator due to the Earth's rotation. $[R_{\text{earth}} = 6400 \text{ km}]$ 8

(c) A 15.0 cm violin string, fixed at both the ends, is vibrating with the largest wavelength. The speed of the wave in this string is 250 m/s, and the speed of sound in air is 350 m/s. What is the frequency and wavelength of the emitted sound wave? 8

(d) Determine the position of focal points, principal points and nodal points for a spherical lens of radius 20 cm. The refractive index of the material of the lens is $3/2$. Indicate the positions of all the points in a diagram. 8

(e) A parallel beam of sodium light of wavelength 5893 \AA is incident on a diffraction grating. The angle between the first order spectra on either side of the normal is $27^\circ 42'$.

Find :

4+4=8

(i) The number of rulings/mm on the grating.

(ii) The greatest number of bright images obtained.

Q2. (a) A particle of mass m is suspended from a spring which has mass m and force constant k . Show that the oscillation frequency is given by $\omega = \frac{\sqrt{3}}{2} \omega_0$, where ω_0 is frequency of oscillation when spring is considered massless. 15

(b) A particle of mass m moves under the attractive central force $F = -\frac{C}{r^{n+1}}$. Find the condition for which the particle will have stable circular orbit. ($C > 0$, a constant). 15

(c) Explain the construction and working of a quarter-wave plate. How is it used to produce circularly and elliptically polarized light? 8+2=10

Q3. (a) (i) Define cyclic coordinates and find their connections with the symmetries of the system.

(ii) A system with two degrees of freedom is described by the Lagrangian $L = \frac{1}{2}m_1\dot{q}_1^2 + \frac{1}{2}m_2q_1^2\dot{q}_2^2 - \frac{\alpha}{q_1}$, α is constant. Find the cyclic coordinates, conserved quantities and symmetries for this system, if any. 10+5=15

(b) Consider two inertial frames S and S' . S' is moving along y -direction with a constant speed v_0 . Find how the first order partial derivatives with respect to x , y , z and ct are connected in these two inertial frames. Show that the D'Alembertian, $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under the Lorentz transformation. 15

(c) Within the framework of the Cornu's spiral, describe the intensity distribution at an arbitrary point P when a plane wave is incident normally on a long narrow slit of width b . For this case, under what condition will the diffraction pattern be of Fraunhofer type? 8+2=10

Q4. (a) Describe the interference by a plane parallel thin film when illuminated normally and obliquely by a plane wave. Also list some important practical applications of the thin film interference phenomenon. 8+2=10

(b) What do you understand by pulse dispersion in multimode optical fibres? Deduce an expression for intermodal pulse dispersion in a step-index fibre. Also compare the pulse dispersions of the step-index fibres with the parabolic-index fibres. 10+5=15

(c) Write down the Euler equations for torque-free motion of a rigid body. Solve these equations to find precessing motion for a symmetric top. 15

SECTION B

Q5. (a) After highlighting the importance of the Biot-Savart law, show that the magnetic field of a current carrying long wire, at a point near it, is inversely proportional to the distance of the point from the wire. 8

(b) A certain linear, homogeneous, isotropic, dielectric material has a relative permittivity, $\epsilon_r = 1.8$. If potential $V = -4000y$ volts in the material, then find :

- (i) The electric flux density D , and
- (ii) The polarisation P .

Take vacuum permittivity $\epsilon_0 = 8.85 \times 10^{-12}$ farad/m.

(c) Consider that an ideal non-interacting Fermi gas with internal energy 'U' at temperature T is kept in a cubical box of volume V . Find the pressure for the gas in terms of U and V . 8

(d) For a plane electromagnetic wave given by :

$$E_z = a \cos \omega x \cos \omega ct$$

$$H_y = -a \sin \omega x \sin \omega ct$$

Find the value of the Poynting vector. 8

(e) For silver, the specific heat at constant pressure in the range of 50 K to 100 K is given by :

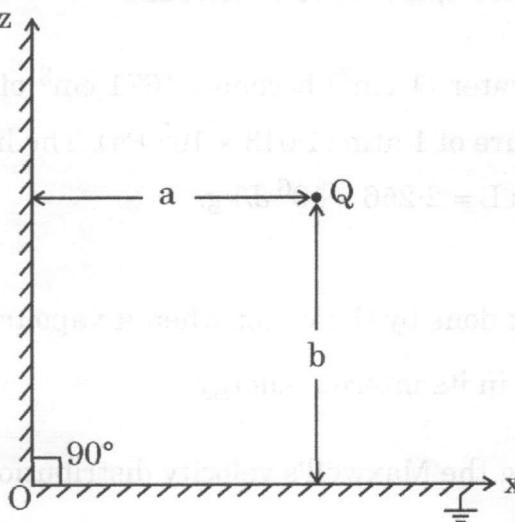
$$C_p = 0.076 T - 0.00026 T^2 - 0.15 \text{ cal mol}^{-1} \text{ deg}^{-1}$$

If 2 moles of silver are heated from 50 K to 100 K, calculate the change in entropy. 8

Q6. (a) If volume charge density in free space varies as $\rho_v = \frac{100\epsilon_0}{r^{2/5}}$, then using Poisson's equation, find potential $V(r)$. It is assumed that $r^2 E_r \rightarrow 0$ when $r \rightarrow 0$, while $V \rightarrow 0$ at $r \rightarrow \infty$. 10

(b) (i) What is the method of images ? What are the conditions which must be satisfied while applying the method of images to deal with electrostatic problems ?

(ii) A point charge Q is located at the point $(a, 0, b)$ between two semi-infinite conducting planes intersecting at right angles as shown in the figure. Using the method of images, determine the potential at point $P(x, y, z)$ in the region $z \geq 0$ and $x \geq 0$ and the force on Q . 5+10=15

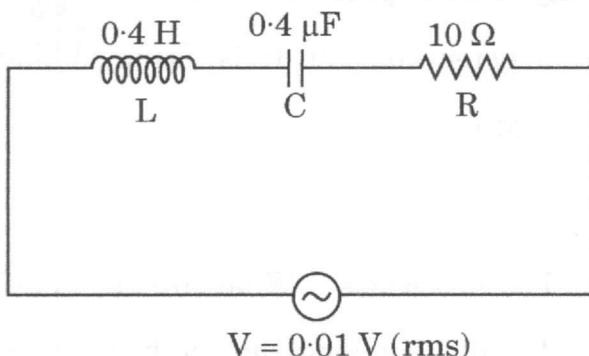


Figure

(c) (i) Show that the vector potential \vec{A} at the position defined by the vector \vec{r} in a uniform electric and magnetic field is $\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r})$. 5+10=15

(ii) Find out the divergence and curl of the vector potential \vec{A} .

Q7. (a) For the following series LCR circuit,



$$V = 0.01 \text{ V (rms)}$$

Calculate :

- (i) The resonant frequency of the circuit.
- (ii) The maximum current in the circuit.
- (iii) The voltage across C at resonance.
- (iv) The power absorbed by the circuit at resonance.

(b) Deduce Fresnel's law for the propagation of plane electromagnetic waves through an anisotropic dielectric medium.

15

15

(c) One gram of water (1 cm^3) becomes 1671 cm^3 of steam when boiled at constant pressure of 1 atm ($1.013 \times 10^5 \text{ Pa}$). The heat of vapourization at this pressure is $L = 2.256 \times 10^6 \text{ J/kg}$.

Calculate :

10

- (i) The work done by the water when it vapourizes, and
- (ii) Increase in its internal energy.

Q8. (a) (i) Assuming the Maxwell's velocity distribution formula, find out the value of :

- (A) Mean velocity (\bar{v}),
- (B) The most probable velocity (v_{mp}), and
- (C) Root mean square speed (v_{rms})

in terms of the Boltzmann constant (k_B) and show that :

$$v_{rms} > \bar{v} > v_{mp}.$$

(ii) If nitrogen molecules are kept at 27°C , find out the value of v_{rms} , \bar{v} and v_{mp} .

Given : Molecular mass of nitrogen $M = 28 \times 10^{-3}$ kg/mol and gas constant $R = 8.314 \text{ J.mol}^{-1}\text{K}^{-1}$. $15+5=20$

(b) Consider N non-interacting ideal spinless particles (Bose gas) are occupying a volume V . Find out the temperature 'T' below which B-E condensation takes place. 10

(c) Assume the Sun to be a black body at temperature 5800 K. Use Stefan's law to calculate :

(i) The total energy emitted by the Sun per second, and
(ii) The energy reaching the top of the Earth's atmosphere.

Given : $\sigma = 5.672 \times 10^{-8}$ SI units,

Radius of the Sun = 7×10^8 m,

The distance of the Earth's atmosphere from the Sun = 1.5×10^{11} m. 10

